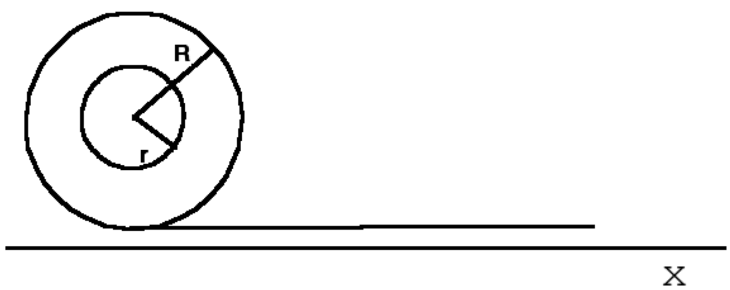
Supplementary calculations for Panomamicam

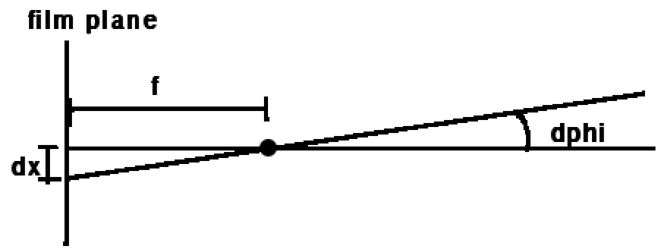
The key challenge with a moving slit camera is in having the film move across the gate at precisely the same rate as the image, so that there is no blur. The most robust system for achieving this digitally would be to continuously monitor the speed of the film directly. However, in designing Panoramicam I couldn't devise an elegant way to track the film speed with a sensor and maintain light-proofness. Instead, I monitor the speed of the gears turning the rewind crank. Since the spool changes diameter as film is wound on, I also have to keep track of how far the spool has turned using "dead reckoning". To achieve all this I started out using some simple analysis of the film and spool system.

First of all, the film is wrapped on a spool, and the effective diameter of the spool increases as the film is wound on. See the following diagram. The variable r is the spool diameter, R the effective spool diameter, t the thickness of the film, and θ the angle the spool has turned.



We need a mathematical relation between the linear speed of the film through the gate and the angular velocity of the rewind spool, so that our electronics can match the film speed to the speed of the moving image. We can see that $\dot{\theta}\cdot R = \dot{x}$. However R is a function of θ, because the film is piling up on the spool. If we consider the wrapped-up film to act about the same as concentric rings of film would, we can say that one rotation of the spool increases R by t, so $R = \frac{t}{2\pi}\cdot\theta + r$, giving the final expression $\dot{\theta}\cdot(\frac{t}{2\pi}\cdot\theta + r) = \dot{x}$. Since our electronics can keep track of the total angle θ by 'dead reckoning' we are off to a good start.

Now we need a mathematical relation between linear film speed $\dot{x}$ and the camera platform speed $\dot{\phi}$, which we know because we control it. The camera is rotating around the nodal point of the lens, which is the lens focal length f away from the film plane.



So we can see that in this case a small change $\Delta\phi$ results in a small image displacement $\Delta x = f \cdot \Delta\phi$ and so $\dot{x} = f \cdot \dot{\phi}$. So to match the rate of image motion to rate of film motion we set the image speed and the film speed equal between this and the above equation and arrive at $\dot{\theta}\cdot(\frac{t}{2\pi} \cdot \theta + r) = f \cdot \dot{\phi}$ which rearranges to $\dot{\theta} = \frac{f}{(\frac{t}{2\pi} \cdot \theta + r)} \cdot \dot{\phi}$. Now, we can put this result in the microcontroller code and, knowing how fast the camera platform is rotating, the focal length of the lens, the thickness of the film, and how far the spool has rotated, our program can 'know' exactly how fast to turn the rewind spool for a sharp, undistorted image. The constants t and r may need to be tweaked slightly from the naively measured starting values, but